HOMEWORK DEPOT STUDY SHEETS

Math11 Formula Sheet

This study sheet provides a basic review of all topics in the BC Math 11 Principles curriculum. For an extensive online lesson of each topic, visit www.BCMath.ca

Ch 2: Quadratic Functions:

I) Quadratic Function:

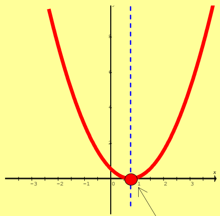
$\boxed{y = a(x-p)^2 + q}$

Vertex: (p,q)

Axis of Symmetry: $\boxed{x = p}$

Domain: $x \in \mathbb{R}$

Range: $y \geq q$ or $y \leq q$



Vertex (p,q)

If 'a' is positive graph opens up

If 'a' is negative graph opens down

Ex: Given $y = 2(x-1)^2 + 3$ **Find all the properties of a parabola:**

$a = 2, p = 1, q = 3$ → Vertex (1,3) (minimum)

Opens Up, AOS: $x = 1$

Domain: $x \in \mathbb{R}$, Range: $y \geq 3$

II) Finding Vertex in General Form:

1st Method: (X.A.V.)

Ex#1) Find the vertex $y = x^2 - 10x + 21$

i) Find X intercepts: (make y = 0)

$0 = (x-3)(x-7) \rightarrow x = 3,7$

ii) Axis of Symmetry (average of x-intercepts)

$\frac{3+7}{2} = 5 \rightarrow \boxed{x = 5}$

iii) Vertex: plug AOS into equation

$y = (5-3)(5-7) \rightarrow y = -4 \rightarrow \boxed{\therefore vertex(5,-4)}$

2nd Method: Find the vertex by Ti-83

$\boxed{2nd}\boxed{Trace}\boxed{3/4 : Max/Min}\boxed{L.Bound}\boxed{R.Bound}\boxed{Guess}$

3rd Method: Completing the Square:

A process to convert a quadratic equation from general to standard form

$\boxed{y = ax^2 + bx + c} \rightarrow \boxed{y = a(x-p)^2 + q}$

Ex: Complete the square: $3x^2 + 12x - 10 = 0$

$3x^2 + 12x - 10 = 0$

$(3x^2 + 12x) - 10 = 0$ bracket 1st 2 terms

$3(x^2 + 4x) - 10 = 0$ factor out "a"

$3(x^2 + 4x + 4 - 4) - 10 = 0$ Add/Subtr. $\left(\frac{1}{2}b\right)^2$

$3(x^2 + 4x + 4) - 12 - 10 = 0$ Take out $-$'ve value, mult w/ "a"

$3(x+2)(x+2) - 22 = 0$ Factor trinomial

$3(x+2)^2 - 22 = 0$ Combine trinom $\rightarrow$ square

$y = 3(x+2)^2 - 22 \rightarrow Vertex(-2,-22)$

iii) Finding the Quadratic Equation:

Ex#3)Find equation of a parabola with vertex (0,3) & passing (3,4)

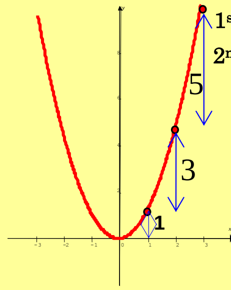
1st step: $p = 0, q = 3, x = 3, y = 4$

2nd step: solve for "a"

$y = a(x-p)^2 + q \rightarrow 4 = a(3-0)^2 + 3$

$a = \frac{1}{9} \rightarrow \boxed{y = \frac{1}{9}x^2 + 3}$

Graphing Quadratic Function: $y = ax^2$



1st Find the vertex from (p,q)

2nd Plot points using 1,3,5,7 process when $a = 1$

If $a = 2$, use 2,6,10,14…

If $a = 3$, use 3,9,15,21…

3rd Graph is symmetrical Flip over the Axis of symmetry for points on other side.

How " a, p, q " Affects the Graph

" p " **Horizontal shift** (Left or right)

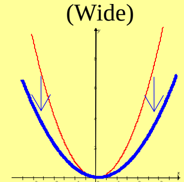
ie: $(x-3)$ 3units right, $(x+2)$ 2units left

" q " **Vertical shift** (up or down)

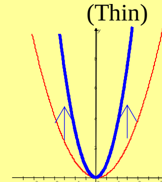
$y = x^2 + 3$ '3 units up' $y = x^2 - 4$ ' 4 units down'

" a " **Vertical expansion or compression**

$\boxed{0 < a < 1}$ (Vert. Comp) (Wide)



$\boxed{a > 1}$ (Vert. Expand) (Thin)



III) Application of Quadratic Functions: Maximizing Revenue

$\boxed{\frac{Q - Q_o}{P - P_o} = \frac{\Delta Q}{\Delta P}}$

$\boxed{R = P \times Q}$

	I	Δ
Q	Q_o	ΔQ
P	P_o	ΔP

Q_o – initial quantity ΔQ – change in quantity

P_o – initial price ΔP – change in price

Ex: A shop sells 400 books at $20 each. Each increase in $4 will result in 40 fewer sales.

a) Write Q as a function of price "p"

b) Find the price that yields maximum Revenue

a)

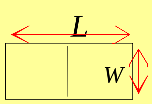
	I	Δ
Q	400	−40
P	20	4

$\rightarrow \frac{Q-400}{P-20} = \frac{-40}{4} \rightarrow Q = 600 - 10p$

b) $R = Q \times P = (600 - 10p)p$ (find vertex)

$Vertex(30, 9000) \rightarrow P = \$30, Max\,Rev = \$9000$

Ex) A farmer wants to build a rectangular barn using 120 meters of fencing separating his cows and chickens. Determine the largest possible area for the barn.



$Perimeter: P = 2L + 3W$

$\rightarrow 120 = 2L + 3W \rightarrow L = 60 - \frac{3}{2}W$

$Area: A = L \times W = \left(60 - \frac{3}{2}W\right)W$

Complete the square to find Vertex (Maximum)

$A = -1.5W^2 + 60W = -1.5(W^2 - 40W)$

$= -1.5(W - 20)^2 + 600 \rightarrow Vertex(20, 600)$

$Width = 20m, Area = 600m^2, L = 30m$

V) Inverse Functions: $y = f^{-1}(x)$

When finding the inverse function, switch "x" & "y", then isolate "y"

Ex: Find the inverse of $f(x) = \frac{3x+1}{4}$

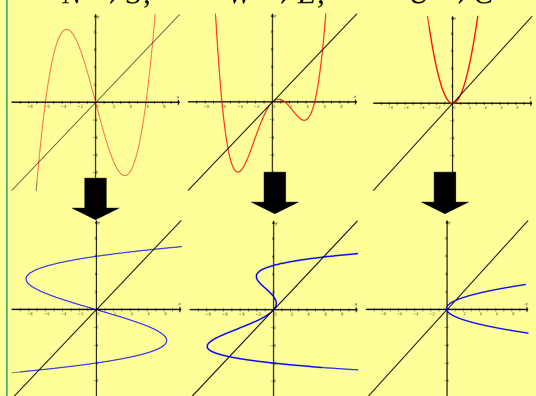
$y = \frac{3x+1}{4} \rightarrow x = \frac{3y+1}{4} \rightarrow 4x = 3y + 1$ (isolate y)

$4x - 1 = 3y \rightarrow \frac{4x-1}{3} = y \rightarrow \boxed{\frac{4x-1}{3} = f^{-1}(x)}$

Graphs of Inverse Functions:

The graph of an Inverse Function is a reflection of $f(x)$ in the line $y = x$

$N \rightarrow S$, $W \rightarrow E$, $U \rightarrow C$



VI) Inverse of Quadratic Function:

When obtaining the inverse of QF, split the domain using the axis of symmetry to 2 parts: The domain of $f(x)$ will become the range of $f^{-1}(x)$:

$\boxed{y = (x-p)^2 + q \rightarrow f^{-1}(x) = \pm\sqrt{x-q} + p}$

$x > p(Right) \rightarrow y > p(Top)$

$x < p(Left) \rightarrow y < p(Bottom)$

Ex: Find the Inverse of $y = (x-2)^2 + 1$

$y = (x-2)^2 + 1$ $\sqrt{x-1} = \sqrt{(y-2)^2}$

$x = (y-2)^2 + 1$ $\pm\sqrt{x-1} = y - 2$

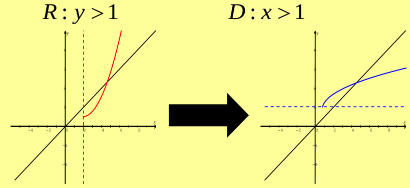
$x - 1 = (y-2)^2$ $2 \pm \sqrt{x-1} = y$

Split the domain to 2 parts: Then $f^{-1}(x)$ is:

$y = (x-2)^2 + 1 \rightarrow \boxed{y = 2 + \sqrt{x-1}}$ &

$D: x > 2$ (Right) $R: y > 2$(Top)

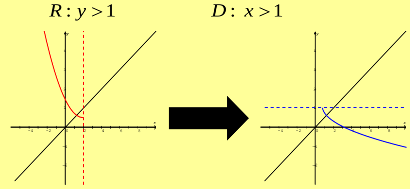
$R: y > 1$ $D: x > 1$



$y = (x-2)^2 + 1 \rightarrow \boxed{y = 2 - \sqrt{x-1}}$

$D: x < 2$ (Left) $R: y < 2(Bottom)$

$R: y > 1$ $D: x > 1$



Ch3: Polynomial & Rational Functions

Polynomial Functions: All powers of x are positive integers & all coefficients are real numbers

$y = ax^n + bx^{n-1} + cx^{n-2} + ... + dx + e$

Linear → $y = mx + b$

Quadratic → $y = ax^2 + bx + c$

Cubic → $y = ax^3 + bx^2 + cx + d$

Quartic → $y = ax^4 + bx^3 + cx^2 + dx + e$

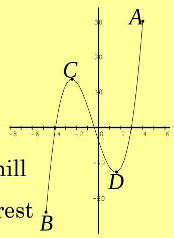
a, b, c, d...are constants

Absolute Max(A): point on graph with the greatest y-coordinate

Absolute Min (B): point on graph with the lowest y-coordinate

Relative Max(C): point on top of a hill

Rel. Min(D): point on bottom of a crest



Note: An Absolute Max/Min may not exist if the graph extends up/down to infinity

Odd Function: Highest degree is Odd
Even Function: Highest degree is even.

3.3: Finding Roots.

Roots, Zeroes, & X-intercepts are all the same thing. This section is about finding the x-intercepts of a function and factoring a function into its factors.

Ex: Find the roots: $y = x^3 + 7x^2 + 12x$

Degree is 3 → 3 possible roots

$0 = x^3 + 7x^2 + 12x$ make y=0

$0 = x(x^2 + 7x + 12)$ factor common coefficient

$0 = x(x+4)(x+3)$ factor trinomial

$\therefore x = 0, -4, -3$

Ex: Write a polynomial function with the following roots:

i) 4,2,−7 → $y = (x-4)(x-2)(x+7)$

ii) 3,3,−5 → $y = (x-3)^2(x+5)$

A double root is when a root appears twice. Graphically, the function will hit the x-axis at the double root and "bounce back".

Solving a Polynomial function.

Ex: Find equation of function with zeroes -1,1,3 and y-intercept at -6

$y = a(x+1)(x-1)(x-3)$ find binomial factors

$-6 = a(0+1)(0-1)(0-3)$ Sub. y-intercept to

$-6 = 3a$ find "a"

$-2 = a \to \boxed{y = -2(x+1)(x-1)(x-3)}$

Missing Constant with Unknown Roots

Ex: Determine the values of "k", so that one root is double the other: $x^2 - kx + 7 = 0$

Note: One root is double the first → r, 2r

$(x-r)(x-2r) = 0$ Expand and Foil

$x^2 - rx - 2rx + 2r^2 = 0$ Simplify

$x^2 - 3rx + 2r^2 = 0$

Compare with original equation:

$x^2 - kx + 7 = 0$ with $x^2 - 3rx + 2r^2 = 0$

$2r^2 = 7 \to r = \pm\sqrt{\frac{7}{2}}$ Solve for "r", then for "k"

$k = 3r \to \boxed{k = \pm 3\sqrt{\frac{7}{2}}}$

Ex: Determine the values of "k", so that one root is triple the other: $16x^2 + kx + 27 = 0$

Note: One root is triple the first → r, 3r

$16x^2 + kx + 27 = 0$

$16(x-r)(x-3r) = 0$

$16(x^2 - rx - 3rx + 3r^2) = 0$

$16x^2 - 64rx + 48r^2 = 0$

$27 = 48r^2 \to r = \pm\frac{3}{4}$

$k = -64r = -64\left(\pm\frac{3}{4}\right)$

$\therefore \boxed{k = \pm 48}$

Ti-83: Finding Zeroes & Intersections

Finding zeroes for polynomial equations:

$\boxed{2nd} \to \boxed{Trace} \to$ "zero" $\boxed{2} \to$ "LeftBound" $\boxed{Enter} \to$ "RightBound" $\boxed{Enter} \to \boxed{Enter}$

Finding Intersection points

$\boxed{2nd} \to \boxed{Trace} \to$ "intersect" $\boxed{5} \to$ "firstcurve" $\boxed{Enter} \to$ "secondcurve" $\boxed{Enter} \to \boxed{Enter}$

Reciprocal Functions: $y = \frac{1}{f(x)}$

3 steps to graphing reciprocal functions

1. *Find zeroes → become vertical asymptotes*
2. *Common* points when "y" co-ord is ±1
3. Large → *small* & small → Large

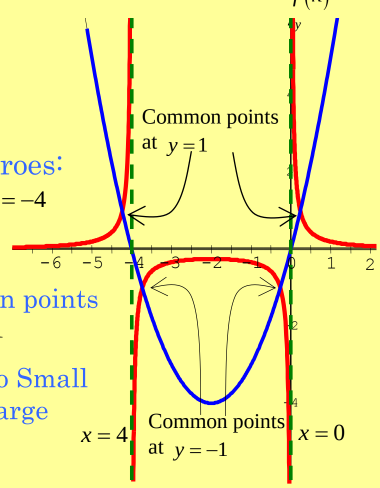
Ex: If $f(x) = (x+2)^2 - 4$ **graph** $y = \frac{1}{f(x)}$

1st graph parabola

2nd find zeroes: (VA) $x = 0, x = -4$

3rd Common points y co-ord $= \pm 1$

4th Large to Small Small to Large



Note:

The recip. of zero is undefined, so we get vertical asymptotes $\frac{1}{0} = undefined$

The recip. of 1 or -1 does not change, so we get common points $\frac{1}{1} = 1$ $\frac{1}{-1} = -1$

The recip. of a large number becomes small and vice versa. $\frac{1}{0.001} = 1000$ $\frac{1}{1000} = 0.001$

Rational Functions (RF):

A RF is a function there is a polynomial in both the numerator & denominator.

$\boxed{Rational\ Function = \frac{Polynomial}{Polynomial}}$

Vertical Asymptote: Values that makes the denominator equal to zero, NPV

Horizontal Asymptote: Use long division. If a remainder exist, the quotient will be the Horizontal Asymptote. If no remainder, then No Horizontal Asymptote.

Ex: $y = \frac{x^2}{x^2 - 9}$ **Find VA, HA, and Graph**

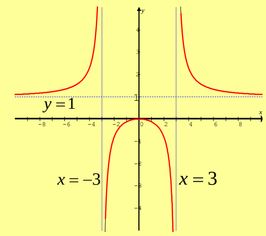
$VA: x^2 - 9 = 0$

$\boxed{x = \pm 3}$

$HA: x^2 - 9\overline{)x^2}$ 1 (quotient)

$-(x^2 - 9)$

$R = 9, \to VA: \boxed{y = 1}$



Ex: $y = \frac{x^2}{x - 5}$ **Find VA, HA, and Graph**

$VA: x - 5 = 0$

$\boxed{x = 5}(NPV)$

$x + 5$(quotient)

$HA: x - 5\overline{)x^2}$

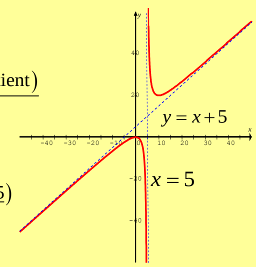
$-(x^2 - 5x)$

$5x$

$(5x - 25)$

R=25

Vert. Asympt. $\boxed{y = x}$



Ex: $y = \frac{2x}{x^2 + 1}$ **Find VA, HA, and Graph**

$VA: x^2 + 1 = 0$

$x^2 = -1$

Can't sq.root!

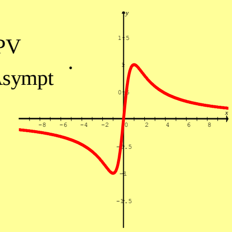
No NPV

No Ver. Asympt

$HA: x^2 + 1\overline{)2x}$

Can't divide

No Vert. Asympt



3.9: Composite Functions:

When 2 or more functions are applied one into another.

Note: Whatever is in the brackets, you replace that for every "x" in the equation.

$f(g(x))$ → Put function G into function F

$g(f(x))$ → Put function F into function G.

$f(x)g(x)$ → Multiply the two functions

Ex#Given $f(x) = 2x - 4$, $g(x) = 3 - 5x$ **Find:**

i) $f(g(x)) =$	ii) $g(f(x)) =$	iii) $g(f(-1)) =$
$= 2(g(x)) - 4$	$= 3 - 5(f(x))$	$= 3 - 5(f(1))$
$= 2(3 - 5x) - 4$	$= 3 - 5(2x - 4)$	$= 3 - 5(-2)$
$= 6 - 10x - 4$	$= 3 - 10x + 20$	$= 3 + 10$
$= 2 - 10x$	$= 23 - 10x$	$= 13$

iv) $f(f(x)) =$	v) $g(g(x)) =$	vi) $g(f(g(x))) =$
$= 2(f(x)) - 4$	$= 3 - 5(g(x))$	$f(g(x)) = 2 - 10x$
$= 2(2x - 4) - 4$	$= 3 - 5(3 - 5x)$	$g(f(g(x)))$
$= 4x - 8 - 4$	$= 3 - 15 + 25x$	$= 3 - 5(2 - 10x)$
$= 4x - 12$	$= -12 + 25x$	$= -7 + 50x$

Ch4:Analysis of Equations & Inequalities

Quadratic Formula:

Formula to solve for "x" with a quadratic equation in the form of $ax^2+bx+c=0$

$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$, where $b^2-4ac\ge 0$

Ex: Find the roots of $2x^2+3x-4=0$

(Find a,b,c then plug into formula)

$a=2, b=3 \to b^2=9, c=-4$

$x=\frac{-3\pm\sqrt{9-4(2)(-4)}}{2(2)} \to x=\frac{-3\pm\sqrt{41}}{4}=\frac{-3\pm 6.403}{4}$

$\boxed{x=0.851 \quad x=-2.35}$

Note: First move all terms to one side and becareful with $\pm$ signs.

4.2: Nature of the Roots:

Use the discriminant formula to find how many roots are in the equation. O, 1 or 2 solns. Discrim Formula: $\boxed{D=b^2-4ac}$

$b^2-4ac>0 \to$ 2 $x-$intercepts

$b^2-4ac=0 \to$ only 1 $x-$intercept

$b^2-4ac<0 \to$ no $x-$intercepts b/c can't $\sqrt{-1}$

Note $\sqrt{b^2-4ac}$ is from Quadratic Formula

Review: Synthetic Division:

1.Use the divisor to find number on the left
2.Bring the first number down, you add downwards
3. Multiple each number on the bottom with divisor to find next number diagonally

Ex#1) Divide $3x^3+11x^2-6x-10$ by $x+4$

$$\begin{array}{r|rrrr} -4 & 3 & 11 & -6 & -10 \\ & \downarrow & -12 & 4 & 8 \\ \hline & 3 & -1 & -2 & -2 \end{array} \to \boxed{Quot: 3x^2-1x-2}$$

$\boxed{\text{Remainder: } -2}$

Note: Degree of terms in dividend must in descending order.

Two Reminders with Synthetic Division

i)If the Dividend is missing a term (skip in the exponents), replace missing term with coefficient of zero ie: (skipped from x^3 to x^1)

Ie: $3x^3-2x+1 \to 3x^3+0x^2-2x+1$

2. If Divisor has a coefficient for the x-term, solve for "x" from the divisor. Do synthetic division. At the end, factor out the same coefficient from the quotient.

Ex: Divide: $4x^3+6x^2-2x+4 \quad \div \quad 2x+1$

$2x+1 \to x=-\frac{1}{2}$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 6 & -2 & 4 \\ & & -2 & -2 & 2 \\ \hline & 4 & 4 & -4 & 6 \end{array} \to$$

$Dividend=\left(x+\frac{1}{2}\right)\left(4x^2+4x-4\right)+6$

D $=2\left(x+\frac{1}{2}\right)\left(2x^2+2x-2\right)+6$

D $=(2x+1)\left(2x^2+2x-2\right)+6$

Quotient (Q): $2x^2+2x-2$ **Divisor (P):** $2x+1$

Dividend $4x^3+6x^2-2x+4$ **Remainder** $R=6$

Division Statement $\boxed{D=PQ+R}$

$\boxed{4x^3+6x^2-2x+4 = (2x+1)\left(2x^2+2x-2\right)+6}$

4.3: Remainder Theorem

When a function $f(x)$ is divided by a binomial $(x-p)$, the remainder will be equal to $f(p)$.

Ex: Find Remainder when $3x^3+11x^2-6x-10$ **is divided by** $(x+4)$**.**

1st Find "p" from divisor $(x+4) \to p=-4$

2nd Substitute "p" into $f(x)$ for "x"

$3(-4)^3+11(-4)^2-6(-4)-10$

$-192+176+24-10=\boxed{-2} \to \text{Remainder}=-2$

Ex: Find "k" when x^3+5x^2+kx-8 **divided by** $(x-3)$**, remainder is 1.**

1st $(x-3) \to p=3$

2nd $f(3)=(3)^3+5(3)^2+k(3)-8=1$

$27+45+3k-8=1 \to -63=3k \to \boxed{k=-21}$

4.4 Factor Theorem:

If you divide a function by binomial $(x-k)$ and the remainder becomes zero, then $(x-k)$ is one of the factors of $f(x)$.

The factor theorem is used to convert a function from General Form to Factored form.

General Form: Factored Form:

$f(x)=2x^3-3x^2-11x+6 \to (x-3)(2x-1)(x+2)$

Steps to Factor a Polynomial:

1. Use Remainder Thm to find 1st root. (Find "x" so that $f(x)$ is zero.) Hint: Use factors of the constant (last) term in dividend.

2.Use Syn. Div. to find quotient. $f(x)$ is the dividend and the root is the divisor. Purpose: break the function $f(x)$ into factors: $(x-k)$

*If quotient has a degree of 3 or bigger, then repeat steps 1&2, until the quotient is a trinomial

3. Factor the quotient (trinomial).

Ex#2)Factor $2x^3+5x^2-x-6$

i)Rem. Thm. Use factors of 6: $\pm1, \pm2, \pm3, \pm6$

$f(1)=2(1)^3+5(1)^2-(1)-6 \to$ Therefore, 1 is a root,

$=2+5-1+6=0$ then (x-1) is a factor

ii) Syn. Div. to find Quotient: Remainder must be Zero!)

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -1 & -6 \\ & & 2 & 7 & 6 \\ \hline & 2 & 7 & 6 & 0 \end{array} \to Quot: 2x^2+7x+6$$

$\to R=0$

iii)Factor the quotient:

$2x^2+7x+6 \to (2x+3)(x+2)$

iv) Solution in Factored Form:

$2x^3+5x^2-x-6 \to \boxed{(2x+3)(x-1)(x+2)}$

Ti-83: Factoring a Polynomial

Type equation into calc. and find the zeroes. The zeroes are the roots.

4.7 :Radical Equations:

Ie: Solve $\sqrt{2x-1}=2-x$

$\left(\sqrt{2x-1}\right)^2=(2-x)^2$ Square both sides

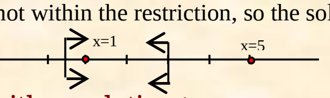
$2x-1=4-4x+x^2$ Restrictions:

$0=x^2-6x+5$ $\quad 2x-1\ge 0 \to \therefore x\ge \frac{1}{2}$

$0=(x-5)(x-1)$ $\quad 2-x\ge 0 \to \therefore 2\ge x$

$x=5, x=1$ Check using number line

$x=5$ is not within the restriction, so the solution is $x=1$



Cases with no solutions:

1. Extraneous roots where solution is not within the restriction
2. Isolate radical and one side is negative

ie: i) $\sqrt{x}=x+3$ ii) $\sqrt{3x-5}=-4$

4.8: Absolute Value $y=|f(x)|$

Steps: Solving $y=|f(x)|$

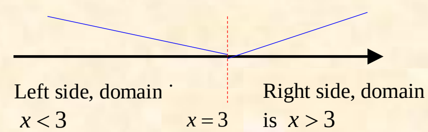
1.Split the graph into 2 domains at the x-int.
2.Draw V-shape graph for $y=|f(x)|$. Label each slope with equation. Left side has a neg. slope & Right has a pos. slope
3. Solve inequality on each side. Check, the solution must be within it's domain.

How to Split the Domain:

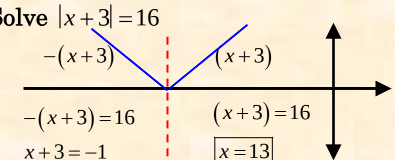
Ie: $|2x-6|=10$

Find x-int. within the abs. value

$2x-6=0 \to x=3$



Ex: Solve $|x+3|=16$



$-(x+3)=16$ $\quad (x+3)=16$

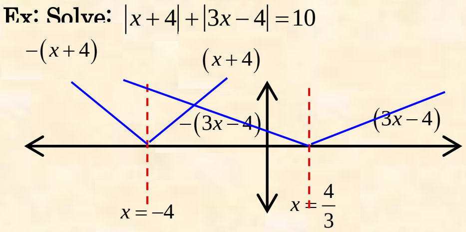
$x+3=-1$ $\quad \boxed{x=13}$

$\boxed{x=-19}$ $\quad x=-3$ $\quad -3>13$ Good

$-19<-3$ Good

The intersections are at $\boxed{x=-19, \; x=13}$

Ex: Solve: $|x+4|+|3x-4|=10$



$-(x+4)-(3x-4)=10$	$(x+4)-(3x-4)=10$	$(x+4)+(3x-4)=10$
$-4x-8=10$	$-2x+8=10$	$4x+4-4=10$
$-4x=-18$	$-2x=2$	$4x=10$
$x=4.5$ (rejected)	$x=-1$(in domain)	$x=2.5$(in domain)

Solutions: $\boxed{x=-1}$ and $\boxed{x=2.5}$

Ch5:Systems of Equations & Inequalities

Review: Line Equations:

$\boxed{y = mx + b}$ m: Slope, b: y-intercept

Ex: Indicate the slope and y-intercept:

$$y = -3x + 4 \rightarrow m = -3, b = 4$$

$$y = 5 - 8x \rightarrow m = -8, b = 5$$

$$y = \frac{2x-8}{4} \rightarrow m = \frac{2}{4} = 0.5, b = \frac{-8}{4} = -2$$

Note: Solving a system means finding the intersection

Solving a Linear System

1st Method 5.3: Solve by Add/Subtr.

Find LCM of coefficients for either "x" or "y". Add/Subtr. to eliminate variable with the same coefficient. Solve for remaining variable.

EX: Solve by Addition or Subtraction

$-7x + 5y = 6 \rightarrow (\times 2) \rightarrow -14x + 10y = 12$

$5x + 2y = 5 \rightarrow \quad (\times 5) \rightarrow \quad 25x + 10y = 25$

Subtract!

$\rightarrow -39x = -13 \rightarrow \boxed{x = \frac{1}{3} \rightarrow y = \frac{5}{3}}$

2nd Method 5.4: Solve by Substitution:

Isolate one variable in 1st equation. Subst. isolated variable into 2nd equation. Then solve.

Ex. Solve by Substitution

$4x + 5y = 13$ Isolate Sub

$x + 2y = 7 \rightarrow \boxed{x = 7 - 2y}$ into 1st eqn!

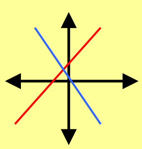
$4(7 - 2y) + 5y = 13 \rightarrow 28 - 8y + 5y = 13$

$-3y = -15 \rightarrow \boxed{y = 5, x = -3}$

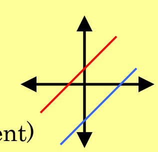
5.2: Number of Solutions in a System

3 Possible Outcomes:

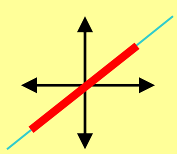
One soln:-1 intersection
Different Slope
(Not Parallel) (Consistent)



No soln:- Parallel
Slope-same but different Y-intercepts
(No intersections – inconsistent)



Infinite soln.: (Same Line)
Slope-same & same y-intercept
(Lines overlap - consistent)



Determining number of solutions in a Linear System with General Form:

When the lines equations are written in general form $ax + by = c$, just compare the coefficients of both equations.

a, b not in ratio $\rightarrow 1$ soln

a, b, c all in ratio $\rightarrow$ infinite

a, b not c in ratio $\rightarrow$ no soln.

Ex: Indicate the number of solution for each system:

a) $3x + 4y = 10 \rightarrow$ *ratio for* a is 2 $\rightarrow a, b$ not in ratio

$6x - 8y = 20$ *ratio for* b is -2 $\therefore$ $\boxed{\text{one soln.}}$

b) $8x - y = 16 \rightarrow$ *ratio for* a, b is $\frac{-3}{4} \rightarrow a, b$ in ratio but

$-6 + \frac{3x}{4}y = 12$ *ratio for* c is $\frac{3}{4}$ not"c" $\therefore$ $\boxed{\text{no soln}}$

c) $x + y = 2 \rightarrow$ *ratio for* a, b, c is $3 \rightarrow a, b, c$ in ratio

$3x + 3y = 6$ $\therefore$ $\boxed{\text{Infinite soln.}}$

5.6 Solving Systems with 3 Variables

Steps:

1. Find LCM for one variable.
2. Add/Subtr. to eliminate variable
3. Use elimination to solve the system

Ex: Solve for: $x, y, \& z$:

$2x - 5y + 2z = 29$ (Coeff. of "x" LCM<2,4,3>=12)

$3x + 6y + 3z = 3$ (Subtr. 1st&2nd eqn to elim. "x")

$4x + 3y + z = 13$ (Subtr. 2nd & 3rd eqn to elim. "x")

$\times 6 \rightarrow 12x - 30y + 12z = 174 \searrow$

$\times 3 \rightarrow 12x + 9y + 3z = 39 \rightarrow$

$\times 4 \rightarrow 12x + 24y + 12z = 12 \nearrow$

$-39y + 9z = 135$

$-15y - 9z = 27$

Add remaining eqn to eliminate "z"

$-54y = 162 \rightarrow \boxed{y = -3}$

$-15(-3) - 9z = 27 \rightarrow \boxed{z = 2}$ Use previous eqn's to solve for "z" then"x"

$3x + 6(-3) + 3(2) = 3 \rightarrow \boxed{x = 5}$

EX: The sum of triple the second and four times the third number is equal to the one plus triple the first number. The sum of the first number and triple the second is equal to 12. The sum of twice the first number, the second number, and five is equal to twice the third. Set up the system of equation.

$3y + 4z = 1 + 3x \quad -3x + 3y + 4z = 1$

$x + 3y = 12 \quad \rightarrow \quad x + 3y + 0 = 12$

$2x + y + 5 = 3z \quad 2x + y - 3z = -5$

5.6) TI-83: Solving Systems with 3 Variables

1st Enter the system into a Matrix:

$\boxed{Matrix}$ → right column $\boxed{Edit}$ → $\boxed{Enter}$

Press 3×4 for a 3 eqn system → enter the coefficients into the Matrix. $\boxed{2nd}$ $\boxed{Quit}$ to exit

2nd Solve the Matrix:

$\boxed{Matrix}$ → right, down to $\boxed{B: rref}$ → $\boxed{Enter}$

$\boxed{Matrix}$ → Scroll down and find the matrix you want to solve then $\boxed{Enter}$

Note Reminders for Inequalities:

If inequality sign is $< or > \rightarrow$ dotted line

If inequality sign is $\leq or \geq \rightarrow$ solid line

Pick a test point that is NOT on line and is easy to evaluate. ie: (0,1), (1,0), (1,1)

If test is True, shade same side

If test if False, shade opposite side

The inequality sign switches sides when you divide or multiply both sides of an equation by a negative number.

5.7 Graphing Linear inequalities:

Steps:

1. Graph the line, $y = mx + b$
2. Pick a test point not on line, ie (0,0)
3. Plug point into line equation, test inequality for True/False
4. True-shade same side, False –other side

Ex: Graph $2x - 5y > 10$

1. Graph $y = \frac{2x}{5} - 2$

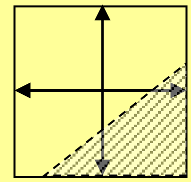
Note: Dotted line

2.. Pick (0,0) to test

$2(0) - 5(0) > 10$

$0 > 10$ False!

3. Test is false, shade other side

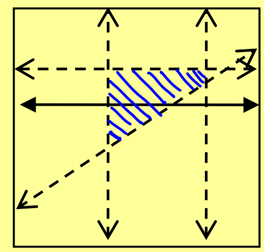


Ex: Graph the following System

$x < 6, \quad x > -1, \quad y < 3,$

$2x - 3y < 6$

Graph each line separately then pick a test point in each inequality (Shade the common side)



Note: $\boxed{x = k}$ is a **vertical line**

$\boxed{y = k}$ is a **horizontal line**

Applications of Linear Systems

Investments: $\boxed{I = Prt}$

I: Interest Earned, P: Principle,
r: rate (decimal), t: Time (years)

Ex: James invested \$1600 into two stocks, A & B. Stock A pays 7% and stock B pays 5%, annually. Total interest earned after one year was \$100. How much was invested in each stock?

1st Set up *Equations*

Interest $A(0.07) + B(0.05) = 100$

Principle $A + B = 1600$

2nd Solve by Elimination

$A(0.07) + B(0.05) = 100 \rightarrow A(0.07) + B(0.05) = 100$

$A + B = 1600 \rightarrow (\times 0.07) \rightarrow A(0.07) + B(0.07) = 112$

→ $0.02B = 12 \rightarrow \boxed{B = \$600, A = \$1000}$

Ex: A car shop makes no more than 10 cars/day, or 15 bikes/day, and no more than 20 vehicles a day altogether. Graph the system

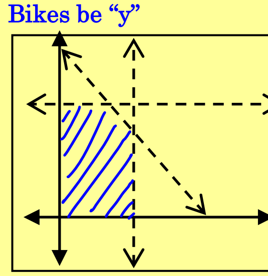
Let: **Cars be "x", Bikes be "y"**

Set up System

$0 \leq x, 0 \leq y$

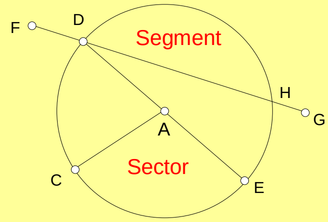
$x \leq 10, y \leq 15$

$x + y \leq 20$



Shaded area is the solution.

Ch 7: Properties of Angles & Chord



Circumference: the distance around a circle $C = \pi d$

Radius: A line with one endpoint is on the circum. and the other on the center of the circle. ie: $\overline{CA}$

Diameter: A line with both endpoints on the circum & the midpoint on the center of the circle. ie: $\overline{DE}$

Chord: Any line with both ends on the circumference. $\overline{DH}$

Arc: a fraction of the circumference:

Major Arc: an arc that is over 50% of the circumference

Minor Arc: an arc that is under 50% of the circumference

Sector: Pizza slice, area b/n two radii's of a circle

Sector Angle: the angle of the sector in the center of the circle $\angle DAC, \angle CAE$

Segment: (watermelon slice) An area of a circle separated by a chord

Secant: An extension of a chord $\overline{FG}$, $\overline{FH}$

Central Angle (aka: Sector angle) angle in the center of circle created by two radii's or diameter. $\angle DAC, \angle CAE$

Inscribed Angle: an angle created by two chords. Angle must be on the circumference. $\angle EDG$

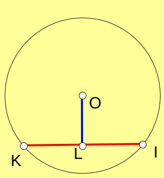
7.2: Chord Properties:

A) A line is perpendicular to a chord (cross at 90°)

B) A line bisects a chord (cut in half)

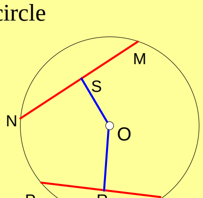
C) A line crosses the center of a circle (line is a radius, diameter, or one endpoint is on center)

If A & B are true → then C must be true:
If A & C are true → then B must be true:
If B & C are true → then A must be true:



ie: If "O" center of circle **(C)** and OL bisects KI **(B)**, then $\angle KLO = 90^\circ$ **(A)**

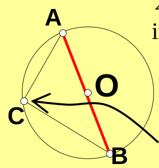
Carpenter's Method: Finding the center of a circle



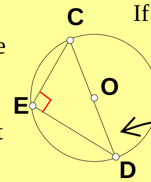
When you have two chords $\overline{NM}$ $\overline{PQ}$ in a circle, the perp. bisectors $\overline{SO}$ $\overline{OR}$ of each chord will cross at the center of the circle.

7.4 Properties of Angles in Circle

1. An inscribed angle in a semi-circle is equal to 90° (aka: an inscribed angle "containing or inscribing" a diameter is equal to 90°)

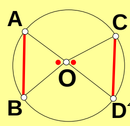


$\angle ACB$ is inscribing the diameter. Therefore, $\angle ACB$ must be 90°

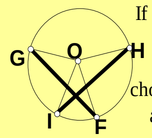


If $\angle CED$ is 90°, then the chord CD, must be the diameter.

2. Central Angles containing equal chords/arcs have equal angles

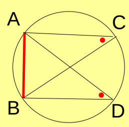


If chords AB and CD have the same length, then $\angle COD = \angle AOB$

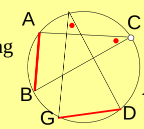


If the central angles $\angle GOI = \angle HOF$ are equal, then the chords $\overline{GI} = \overline{HF}$ are also equal.

3. Inscribed angles containing(subtending) the same(equal) chord/arc, will have equal angles

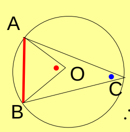


$\angle ACB$ and $\angle AOB$ are both inscribing chord AB, so $\angle ACB = \angle AOB$

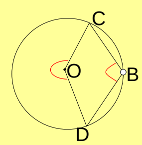


If chords AB = GD then $\angle AFB = \angle GCD$

4. The inscribed angle is equal to half of the central angle containing(subtending) the same chord/arc.

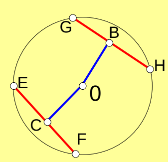


$\angle AOB$ is a central angle, $\angle ACB$ is an inscribed angle. Both contain chord AB.
$\therefore \angle AOB = 2 \times \angle ACB$.
$\frac{1}{2}\angle AOB = \angle ACB$



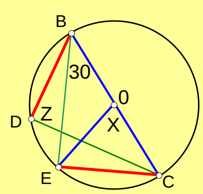
The outer angle $\angle COD$ is equal to twice of $\angle CBD$.
Then $2 \times \angle CBD = \angle COD$

5. **Two Chord Theorem**



If two chords in a circle have the same length, they are equal distance from the center.
If $\overline{GH} = \overline{EF}$ → $\overline{CO} = \overline{BO}$
If two chords are equal distance from center, the chords have the same length.

Ex: Given "O" is the center and EC=6cm, find measure of $\angle x, \angle z$, and $\overline{BC}$.



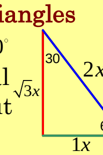
$\angle x = 60$; $centr\angle = 2 \times inscr\angle$

$\angle z = 90^\circ$; $inscr\angle$ by diameter

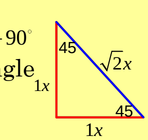
$\overline{BC} = 12$; $\triangle$BEC is a 30-60-90 special triangle

Special Triangles

$30^\circ - 60^\circ - 90^\circ$
Equilateral triangle cut in half



$45^\circ - 45^\circ - 90^\circ$
Right angle isosceles triangle



Pythagorean Triples: $\boxed{a^2 + b^2 = c^2}$

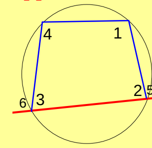
3,4,5 5,12,13 8,15,17, 7,24,25 20,21,29 9,40,41
6,8,10 10,24,26 16,30,34 14,28,50

NOTE: Many questions in this chapter contain right triangles. Use Pythagorean triples or Special triangles to find lengths of missing sides.

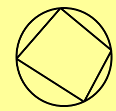
Ch 8: Other Circle Properties:

8.1 Properties of a Cyclic Quadrilateral CQ:

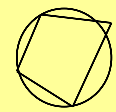
i) All 4 vertices(corners) must be on the circumference of the circle
ii) Opposite angles in a CQ add to 180°
iii) The Exterior Angle of a CQ is equal to the opposite interior angle



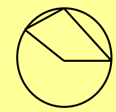
Property ii)
$\angle 1$ & $\angle 3$ are opp.∠'s → $\angle 1 + \angle 3 = 180^\circ$
$\angle 2$ & $\angle 4$ are opp. ∠'s → $\angle 2 + \angle 4 = 180^\circ$
Property iii)
$\angle 5$ is ext∠, $\angle 4$ is opp. int∠, $\therefore \angle 5 = \angle 4$
$\angle 6$ is ext∠, $\angle 1$ is opp. int∠, $\therefore \angle 6 = \angle 1$



IS a CQ!



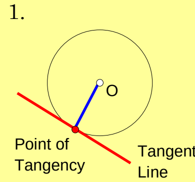
Not a CQ! One corner not on circumference



Not a CQ! Corner is on center

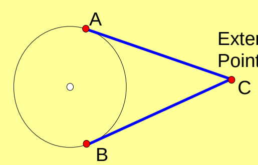
Tangent Properties:

1.



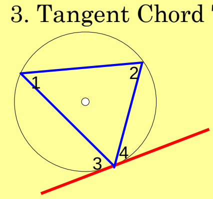
A tangent line to a circle is perp. to the radius at the point of tangency.
Note: A tangent line will intersect a circle at only one

2.



Tangent points to a similar exterior point are equal in length: $\overline{AC} = \overline{BC}$

3. Tangent Chord Theorem



Angle between a tangent line and chord of a circle is equal to the inscribed angle $\angle 1 = \angle 4$ & $\angle 2 = \angle 3$

8.6 Angles in a Polygon:

i) Sum of all angles in a polygon with "n" sides: $\boxed{S = 180^\circ(n-2)}$

ii) Value of each interior angle in a Regular polygon with "n" sides. $\boxed{A = 180 - \frac{360}{n}}$

Ex: Find the sum of all the interior angles in a 5 pointed star.

10 sides!
$n = 10 \to S = 180(10-2) = 1440^\circ$

Author: D.G.W. Young
Made in Canada

Ch 9: Co-Ordinate Geometry

Review Formulas:

Slope: $\boxed{\frac{y_2-y_1}{x_2-x_1}=\frac{Rise}{Run}}$

Midpoint: $\boxed{(x_m, y_m)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)}$

Distance: $\boxed{D=\sqrt{(x_2-x_1)^2+(y_1-y_2)^2}}$

Slopes of perpendicular lines are negative reciprocals. $\boxed{m \to \frac{1}{-m}}$ (Flip the number and change the sign. $ie: 3 \to \frac{1}{-3}$

Multiplying slopes of two perpendicular lines will become -1.

Trigonometry: SOH-CAH-TOA

$\boxed{\sin x=\frac{opp}{hyp}}$ $\boxed{\cos x=\frac{adj}{hyp}}$ $\boxed{\tan x=\frac{opp}{adj}}$

9.1 Circle Equation:

$\boxed{R^2=(x-h)^2+(y-k)^2}$

Radius: R Center at (h,k)

h: horizontal shift, k: vertical shift

Ex Find the equation of a circle with radius 5 and center at $(3,-5)$

R=5, h=3, k=-5 → $25=(x-2)^2+(y+5)^2$

Ex Find the equation of a circle with endpoints of a diameter at $(-2,4)(6,-6)$

1st Use Midpoint → find Center

$(x_m, y_m)=\left(\frac{-2+6}{2},\frac{4+(-6)}{2}\right)=(2,-1)\to(h,k)$

2nd Use Distance → find Radius

Center: $(2,-1)$ Endpoint: $(-2,4)$ $(6,-6)$

$D=\sqrt{(2-6)^2+(-1-(-6))^2}=\sqrt{41}\to R$

3rd Substitute into circle equation:

$R=\sqrt{41}\to R^2=41$ $41=(x-2)^2+(y+1)^2$

Graphing Circles:

1st: Find the center (h,k)

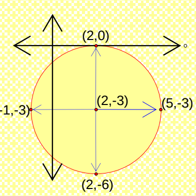
2nd: Find the radius: R

3rd: Draw points "R" units from the center and connect the points (up, down, left, right)

Ex: Graph $(x-2)^2+(y+3)^2=9$

Center $(2,-3)$

Radius = 3



Note: When h,k are both zero, then the center is at the origin. Ie: $x^2+y^2=9$

To find more points on the circle, create a right triangle with the radius as the hypotenuse. Use Pythagorean Triples if possible.

9.2: Lines & Circles

Ex: Find the intersections points of

$(x+1)^2+(y-3)^2=9$ & $y=x+1$

$(x+1)^2+(y-3)^2=9$ 1st Substitution: $y\to x+1$

$(x+1)^2+(x+1-3)^2=9$ FOIL

$x^2+2x+1+x^2-4x+4=9$ 2nd Solve for "x"

$2x^2-2x-4=0$ Factor trinomial

$2(x-2)(x+1)=0$

$\to x=2, x=-1$ 3rd Solve for "y"

$y=3, y=0$ → Points of intersection are $\boxed{(2,3)(-1,0)}$

Ex: Find the equation of a tangent line to the circle $(x-2)^2+(y-3)^2=13$ **at the point** $(5,1)$

Note: Slope of radius is perpendicular to the tangent line

1st Find slope of radius from center to tangent point

$(radius)\, m=\frac{3-1}{2-5}=\frac{2}{-3}$

2nd Find slope of tangent line (perpendicular)

$\to m=\frac{3}{2}(neg.\ reciprocals)$

3rd Find equation of tangent line: point $(5,1)$, $m=\frac{3}{2}$

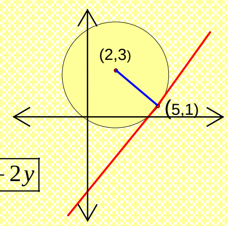
$y=\frac{3}{2}x+b$

$1=\frac{3}{2}(5)+b$

$\frac{-13}{2}=b$

$\boxed{y=\frac{3}{2}x-\frac{13}{2}}$ or

$2y=3x-13\to\boxed{13=3x-2y}$



Shortest Distance Problems:

The shortest distance from a point to a line is always perpendicular to the line

1st Method: Trigonometry:

Ex: Find the shortest distance from the origin to the line: $3x+5y=15$

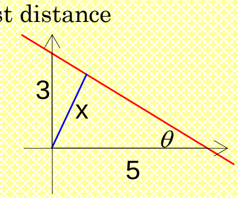
Let "x" be the shortest distance

1st: use $\tan^{-1}$ to find θ

$\tan\theta=\frac{3}{5}\to\theta=59^\circ$

2nd: use sin to find "x"

$\sin\theta=\frac{x}{5}\to x=2.57$



2nd Method: Shortest Distance Formula

The shortest distance from any point (m,n) to any line in the form of $Ax+By+C=0$ is:

$\boxed{d=\frac{|A(m)+B(m)+C|}{\sqrt{A^2+B^2}}}$ *Some teachers prohibit the use of this formula

Ex: Find the shortest distance from the point $(4,-3)$ **to the line:** $3x+5y=15$

1st Find (m,n) & ABC $m=4, n=-3$

$3x+5y=15\to3x+5y-15=0$

$A=3, B=5, C=-15$

2nd: Substitute into equation:

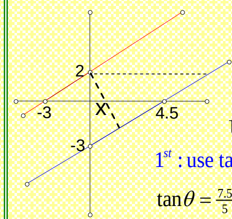
$d=\frac{|3(4)+5(-3)+(-15)|}{\sqrt{3^2+5^2}}=\frac{|-18|}{\sqrt{34}}=3.087$

Shortest Distance Between Two Parallel Lines

Vertical Distance: Difference of y-intercepts

Horizontal Dist: Difference of x-intercepts

Ex: Find the Vertical, Horizontal, & Shortest Distance: $2x-3y=9$ **&** $2x-3y=-6$



Hor. $-3-4.5=-7.5$

Vert. $2-(-3)=5$

Shortest Distance:

Use Right Triangle & Trig.

1st: use $\tan^{-1}$ to find θ (angle at the bottom)

$\tan\theta=\frac{7.5}{5}\to\theta=56^\circ$

2nd: use sin to find "x"

$\sin\theta=\frac{x}{5}\to x=4.16$

Ex: Find the equation of the perpendicular bisector of the line with endpoints $(2,-9)\&(4,6)$

1stFind slope & Midpoint of line with endpoints

$m=\frac{-9-6}{2-4}=\frac{-15}{-2}=\frac{15}{2}$ Midpt: $\left(\frac{2+4}{2},\frac{-9+6}{2}\right)=(3,-1.5)$

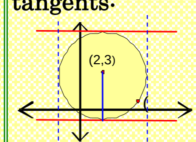
2nd Find equation of perpendicular bisector:

$y=mx+b$: $m=\frac{-2}{15}$ (perpendicular) Point $(3,-1.5)$

$-1.5=\frac{-2}{15}(3)+b$

$-1.1=b$ $\to\boxed{y=\frac{-2}{15}x-1.1}$

Ex: Given $(x-2)^2+(y+3)^2=16$**, find the equation of the vertical and horizontal tangents:**



Horizontal Tangents:

$y=1$ $y=-7$

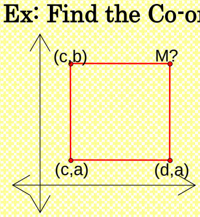
Vertical Tangents:

$x=6$ $x=-2$

9.5 Using Co-Ordinate Geometry to Prove Conjectures

Variables are used to represent numbers when labelling co-ordinates

Ex: Find the Co-ordinates of M.

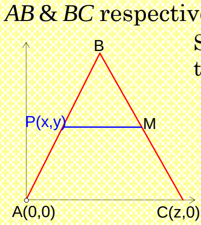


x-coord is "d"

y-coord is "b"

Answer: $M(d,b)$

Ex: Find the coordinates of 'M' given that both "P" & "M" are both midpoints of $\overline{AB}$ & $\overline{BC}$ respectively.



Since "P" is the midpoint of AB, the co-ord. of "B" is (2x,2y)

Co-ord of "M":

Midpoint-B(2x,2y) C(z,0)

$\left(\frac{2x+z}{2},\frac{2y}{2}\right)=\left(x+\frac{1}{2}z,y\right)$

b) Prove that $\overline{PM}=\frac{1}{2}\overline{AC}$

Dist. of AC = z, so $\frac{1}{2}AC=\boxed{\frac{1}{2}z}$

Dist. of PM= difference of x-coordinates

$=\left(x+\frac{1}{2}z\right)-(x)=\boxed{\frac{1}{2}z}$

Therefore, $\overline{PM}=\frac{1}{2}\overline{AC}=\frac{1}{2}z$